

Mathematics

Accomplished teachers have a command of the mathematical content knowledge required to teach students ages 7–12 and beyond. They know and understand the major concepts and procedures that define number, including operations and the problems they solve, geometry, measurement, and statistics and probability. They are also cognizant of algebraic thinking topics that include work with patterns, expressions and equations, functions, and ratio and proportion. Teachers are well versed in important mathematical practices, particularly the critical role of problem solving and reasoning in mathematics teaching and learning. They routinely stress the importance of problem solving and encourage their students to make sense of the problems they are solving and to persevere in solving them.

Accomplished teachers recognize that an important, albeit sometimes unwritten, expectation of mathematics learning at these levels is the development of number sense. This instructional trajectory begins with counting and advances to include place value, operations, mental mathematics, and estimation, all directed toward computational fluency with whole numbers and fractions (defined here as a/b fractions, decimals, and common percents). Mathematics at these levels includes important geometric concepts and vocabulary involving shape, as well as applications involving measurement, such as perimeter, area, and volume. Problem-based contexts provide students with opportunities to gather and analyze data. Teachers at these levels also know that much of their work with early algebra, including expressions, equations, and functions, serve as foundational building blocks to more formal experiences in algebra at the middle and high school levels.

Accomplished teachers comprehend how knowledge in mathematics is organized and sequenced and how it is connected to other subject areas. Teachers recognize that how children learn mathematics and which mathematics children should learn are topics of ongoing research. They know how to build on children's informal understandings as a link to more formal mathematics. For example, they may share eight objects among three students to have them think about how to use division and to help them understand fractions and mixed numbers. Accomplished teachers

know that mathematics learning is critical. They serve as advocates to ensure that all students have access to meaningful mathematics instruction, which is instrumental for achieving success in other subject areas.

In the classrooms of accomplished teachers, students are engaged through a variety of activities that reinforce the concepts and skills they are learning. For example, a teacher may provide student groups with 3 meters of yarn and ask them to create shapes and determine the shape with the greatest area. Another activity would be to have students determine the best bargain for buying a beverage: five bottles for \$12 or two bottles for \$5. Teachers encourage students to explain their thinking by discussing their reasoning. Students thereby construct viable arguments by collaborating and interacting with one another as they solve problems and discover the impact of mathematics on their lives. Accomplished teachers help students recognize that there are multiple ways to solve many problems and that thinking through and explaining a problem is as critical as providing the correct solution.

Recognizing the significance of mathematical language, accomplished teachers provide students with ways to link mathematical vocabulary to important expectations. For example, teachers may have students use journals to record mathematical vocabulary using words, pictured representations, or equations. An entry for the distributive property might appear as follows: $23 \times 8 = (20 \times 8) + (3 \times 8) = 160 + 24$, and could also include a visual representation using a bar or box diagram.

Accomplished teachers use classroom resources in an effective manner. These resources include, but are not limited to, textbooks, supplemental resources, manipulative models and materials—hands-on and virtual—and technological tools. They are comfortable using a variety of manipulative and pictorial representations to establish conceptual understandings and help students connect concepts to procedures. For instance, teachers might have students use base 10 blocks to represent tenths and hundredths when learning about decimals or use a number line or rectangular regions when comparing fractions. In addition, accomplished teachers can readily locate or create problem solving tasks that challenge their students and extend their mathematics learning to higher-level concepts or connect them to other mathematical topics and subject areas. Teachers are acutely aware of the need to prepare students for a more global and technologically-rich society. They understand and use emerging technological tools to expand and enhance their pedagogy accordingly. These tools may include Web sites, virtual models and manipulatives, online texts, or spreadsheet applications.

Adept at diagnosing learner needs in mathematics, accomplished teachers provide interventions and accommodations accordingly. They recognize and value the different ways that students think and interpret mathematics, and they take these differences into consideration when creating classroom learning environments. These environments support and encourage the development of mathematical practices that include making sense of problems, reasoning through mathematical situations, and constructing viable arguments. Importantly, accomplished teachers realize that an approach to mathematics learning that is engaging, challenging, and problem-

based is critical for their students to develop and maintain a positive disposition toward mathematics.